# ME-221
# PROBLEM SET 2

## Problem 1

Obtain mathematical models of the spring-mass systems shown in Figure 1 and derive the equivalent spring constant for parallel and serial arrangements. The input to the systems is the force $F$, the displacement $x$ is measured from the equilibrium position, and the spring coefficients are given by $k$, $k_1$, and $k_2$.

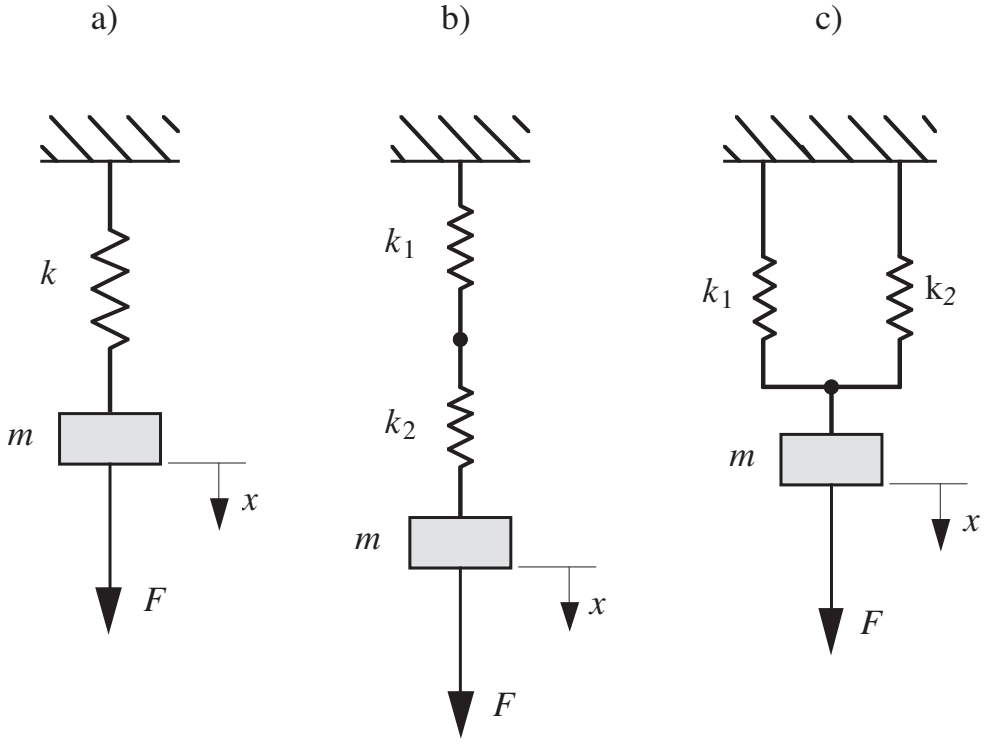


Figure 1: Spring-mass systems

## Problem 2

Derive the equations of motion for the lever system shown in Figure 2. The input to the systems is the vertical force $F$, and the displacement $x$ is measured from the equilibrium position A. The spring and viscous friction coefficients are given by $k$ and $f$, respectively. Comment on the assumptions you have made to simplify the model.

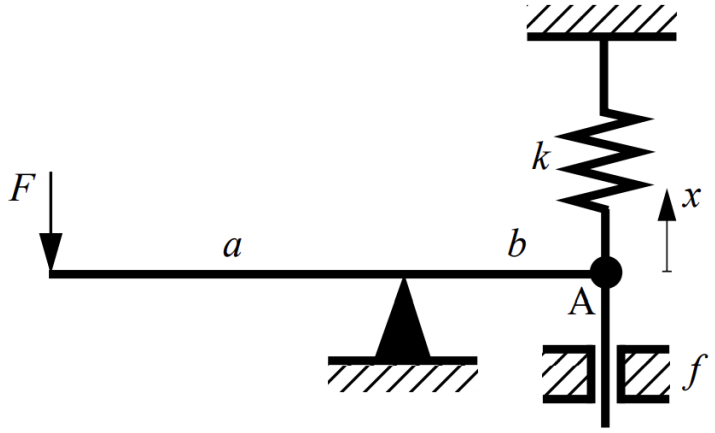


Figure 2: Lever arm connected to a spring mechanism

# Problem 3

The system in Figure 3 represents an accelerometer capable of measuring the acceleration of the object on which it is attached. $y$ is the displacement of the object whose displacement we want to measure. $z$ is the displacement of the mass $m$ with respect to the housing of the accelerometer.

a) Derive a mathematical model of the mechanical system given in Figure 3. The input to the system is the acceleration $\ddot{y}$ and the output is the displacement $z$ of the mass $m$ with respect to the frame of the accelerometer. The spring and viscous friction coefficients are given by $k$ and $f$, respectively. Hint: Since Newton's law of motion must be applied with respect to absolute displacements, define $x$ as the displacement of $m$ relative to a fixed landmark, as shown in Figure 3.

b) In which case is the acceleration $\ddot{y}$ proportional to displacement $z$?

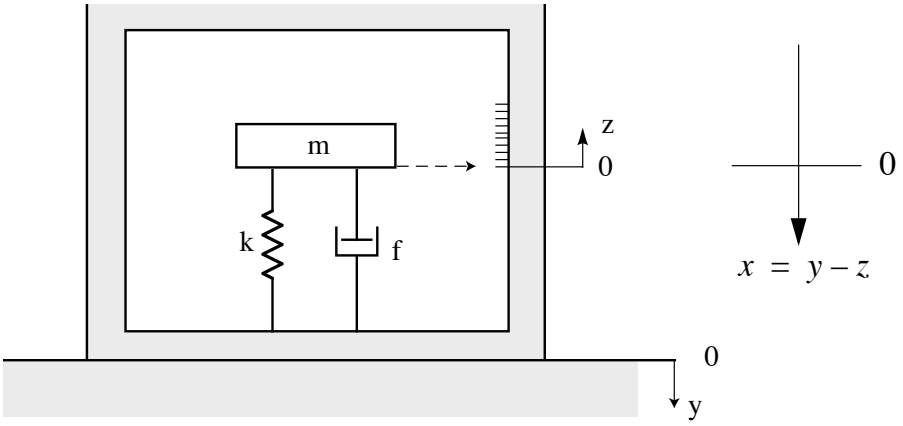


Figure 3: Spring-mass-damper system with accelerometer

# Problem 4

Obtain a mathematical model for the electrical circuit shown in Figure 4. The input to the systems is the voltage $u$ and the voltage $y$ is measured as the output. The capacitance $C$ is given by $C = \alpha_0 + \alpha_1 y$, where $\alpha_1$ and $\alpha_2$ are constant parameters. Hint: Choose the voltage on the capacitor and the current passing through the inductor as the system states.

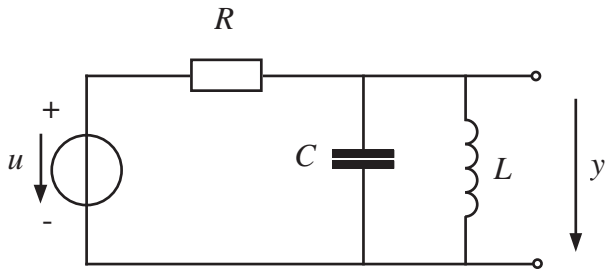


Figure 4: RLC circuit

# Problem 5

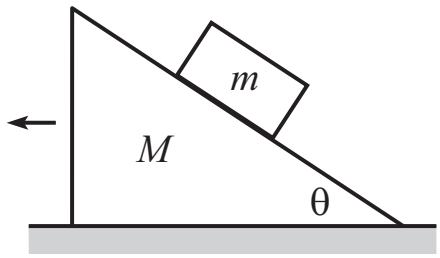


Figure 5: Moving plane

A block of mass $m$ is held motionless on the frictionless inclined plane of a wedge of mass $M$ and angle of inclination $\theta$ as shown in Figure 5. The wedge rests on a frictionless horizontal surface. The block is released. What is the horizontal acceleration of the wedge (which is expected to be in the direction of the arrow)? Hint: If you are missing an equation to solve your system, try to come up with one which forces the block to stay in contact with the wedge.